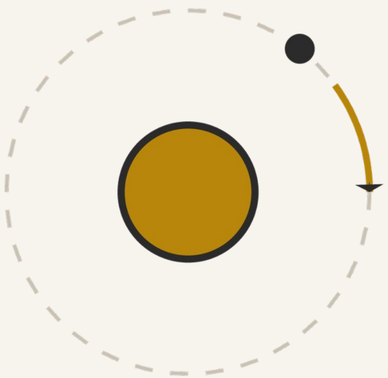
ROTATIONAL MOTION AND ASTROPHYSICS · 1.4 GRAVITATION

Gravitation

Every mass pulls on every other — and the maths reaches across the solar system.



Advanced Higher Physics

LEARNING INTENTION

To describe gravitational fields and potential, and use them to solve problems about orbits and escape.

By the end of this topic…

1. I can convert between AU, light-years and metres.
2. I can state the definitions of gravitational field strength, potential and escape velocity.
3. I can sketch gravitational field lines for one mass and for two.
4. I can solve satellite problems using $GMm/r^2 = mv^2/r = mr\omega^2$.
5. I can derive $v_{esc} = \sqrt{(2GM/r)}$ and use it.

PERIOD 1 · THE LAW AND THE FIELD

What does a mass do to space?

Every other mass feels it.

JOTTER NOTES

Distances in space

Space is too big for metres, so two other units are used.

1 AU is the Earth-Sun distance, $1\,\mathrm{AU} = 1{\cdot}5 \times 10^{11}\,\mathrm{m}$.

1 light-year is how far light travels in a year:

$1\,\mathrm{ly} = 3{\cdot}0 \times 10^{8} \times 365 \times 24 \times 60 \times 60 = 9{\cdot}46 \times 10^{15}\,\mathrm{m}$.

Always convert to metres before using any relationship.

JOTTER NOTES

Newton's universal law of gravitation

Every mass attracts every other mass:

$$F = \frac{GMm}{r^2}$$

M and m are the two masses and r is the distance between their centres.

$G = 6{\cdot}67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$, the gravitational constant.

G is a constant everywhere; g is a field strength and varies.

Gravitational field strength

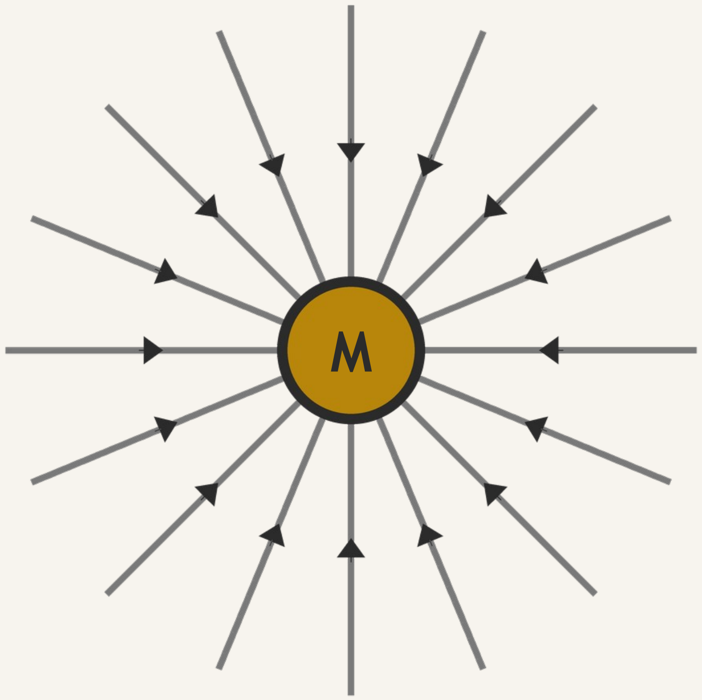
JOTTER NOTES

LEARN THIS WORD FOR WORD

The gravitational field strength at a point is the gravitational **force acting on a unit mass** placed at that point. $g = F/m$

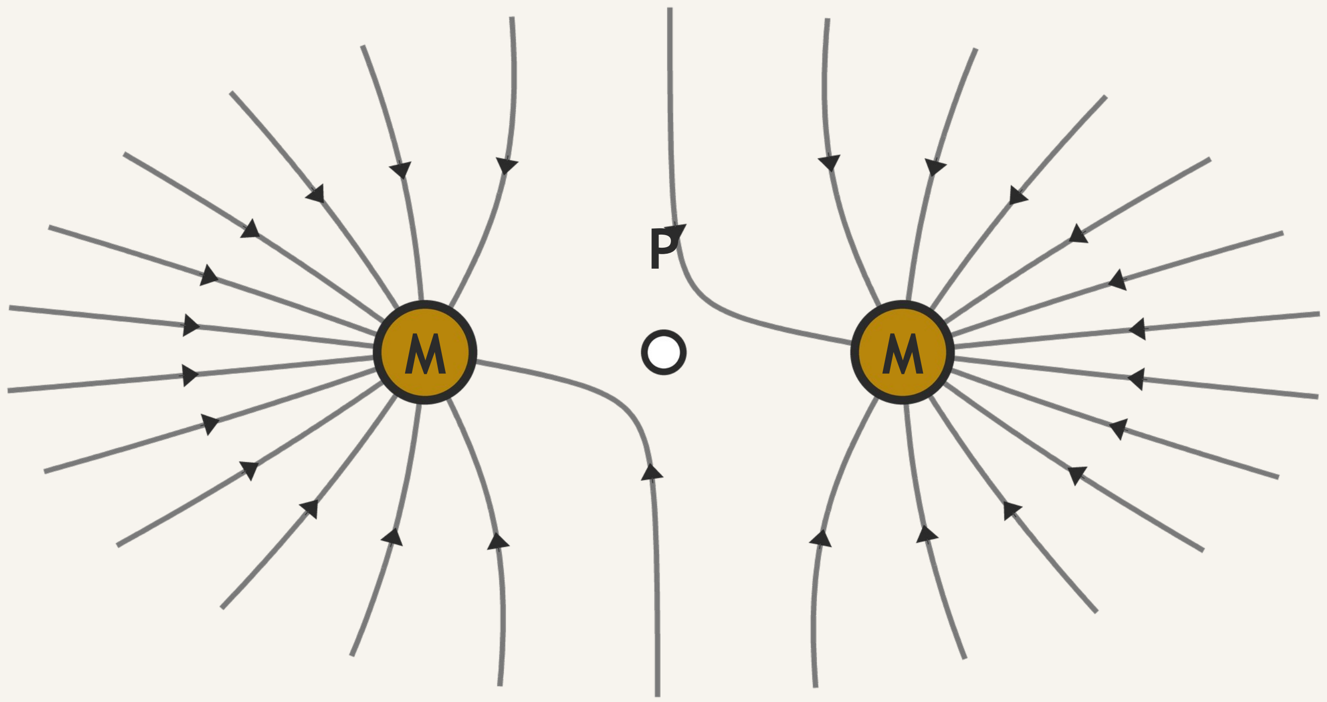
The field around a single mass

Field lines point the way the force on a mass would act



The field around two masses

Between them the two fields cancel — that is the neutral point



At P the two pulls balance, so the field is zero.

Starter — from memory

Close your notes. Write full sentences.

1. State the definition of gravitational field strength.
2. Convert 1 light-year into metres.
3. State Newton's universal law of gravitation.
4. State the principle of conservation of angular momentum.
5. Define the moment of inertia of an object.

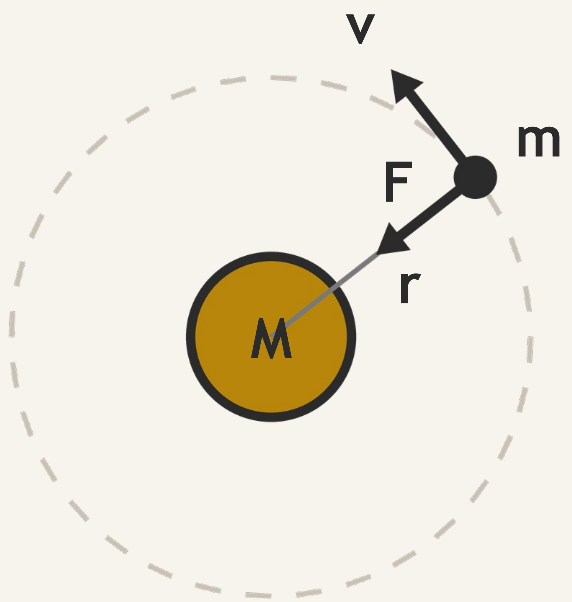
PERIOD 2 · SATELLITES IN ORBIT

What holds a satellite up?

Nothing. It is falling, and missing.

Gravity is the centripetal force

The satellite is not held up — it is pulled inwards, and travels sideways



JOTTER NOTES

The satellite relationship

The gravitational force provides the centripetal force, so

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2$$

and since ω = 2π/T,

$$mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

Worked example 1 — 2018 Q4(a)(i)

Exoplanet d orbits the star HD 69830 in a circular orbit of radius r.

Show that its period T is given by $T^2 = 4\pi^2 r^3 / GM$.

orbit is circular

star mass = M

orbit radius = r

show $T^2 = 4\pi^2 r^3 / GM$

$$F_{centripetal} = F_{gravitational}$$

$$mr\omega^2 = \frac{GMm}{r^2}$$

$$\omega = \frac{2\pi}{T}$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Worked example 2 — 2018 Q4(a)(ii)

Exoplanet d has a mean orbital radius of 0·63 AU and an orbital period of 197 Earth days. Find the mass of the star HD 69830.

r = 0·63 AU

T = 197 days

M = ?

$r = 0{\cdot}63 \times 1{\cdot}5 \times 10^{11} = 9{\cdot}45 \times 10^{10}\ \text{m}$

$T = 197 \times 24 \times 60 \times 60 = 1{\cdot}70 \times 10^{7}\ \text{s}$

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$\underline{M = 1{\cdot}7 \times 10^{30}\ \text{kg}}$

Satellites in real use

The orbit you choose follows from the job it has to do

Satellite	Altitude	Period	What it is for
Geostationary	35 800 km	24 h	TV, telecomms
Polar, low Earth	600–800 km	100 min	weather
GPS	20 200 km	12 h	navigation
Space Station	400 km	93 min	research

Starter — from memory

Close your notes. Write full sentences.

1. State the definition of gravitational field strength.
2. Write the relationship linking gravitational force, masses and distance.
3. A satellite orbits at radius r. What supplies its centripetal force?
4. Convert 0·63 AU into metres.
5. State Newton's universal law of gravitation.

PERIOD 3 · GRAVITATIONAL POTENTIAL

How much work to get away?

Measured from infinity, and always negative.

Gravitational potential

JOTTER NOTES

LEARN THIS WORD FOR WORD

The gravitational potential at a point is the work done in moving **unit mass from infinity** to that point.   Measured in $J\ kg^{-1}$.

JOTTER NOTES

Why the potential is always negative

Potential is defined as zero at infinity, infinitely far away.

Moving a mass in from infinity, the field does the work, not you.

So the mass ends up with less energy than it started with,

and every real point in a field has a negative potential.

The closer to the mass, the more negative the potential.

JOTTER NOTES

Potential and potential energy

The potential at distance r from a mass M is

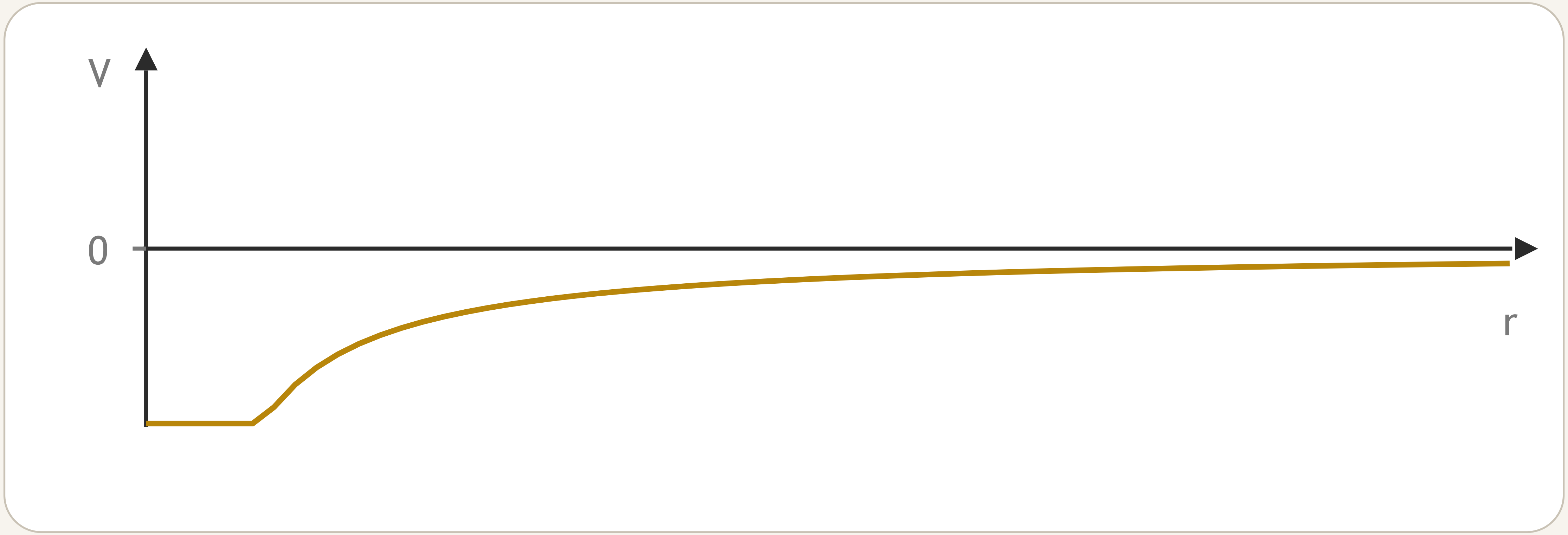
$$V = -\frac{GM}{r}$$

and a mass m placed at that point has potential energy

$$E_p = Vm = -\frac{GMm}{r}$$

Potential against distance

V rises towards zero as r grows, and only reaches it at infinity



Potential is not potential energy

Gravitational potential V	Gravitational potential energy E{_p}
a property of the point, not of any mass	a property of a mass at that point
measured in $J\ kg^{-1}$	measured in J; doubles if m doubles
$V = -GM/r$	$E_p = Vm = -GMm/r$

JOTTER NOTES

The path does not matter

The work done moving a mass between two points in a gravitational field does not depend on the route taken — only on the two points.

A field like this is called a conservative field.

Move a mass round any closed path and the net work done is zero.

Worked example 3 — 2019 Q5(b)

The Juno spacecraft is $1{\cdot}69 \times 10^{8}$ m from the centre of Jupiter, whose mass is $1{\cdot}90 \times 10^{27}$ kg. Find the gravitational potential there.

$r = 1{\cdot}69 \times 10^{8}\ \mathrm{m}$

$M = 1{\cdot}90 \times 10^{27}\ \mathrm{kg}$

$V = ?$

$$V = -\frac{GM}{r}$$

$$V = -\frac{6 \cdot 67 \times 10^{-11} \times 1 \cdot 90 \times 10^{27}}{1 \cdot 69 \times 10^{8}}$$

$\underline{V = -7{\cdot}50 \times 10^{8}\ \mathrm{J\ kg^{-1}}}$

Worked example 4 — 2019 Q5(c)

Juno has mass $1{\cdot}6 \times 10^3$ kg. It moves from point A, where the potential is $-1{\cdot}70 \times 10^9$ J kg^{-1}, to point B above. Find ΔE_p.

$m = 1{\cdot}6 \times 10^3$ kg

$V_A = -1{\cdot}70 \times 10^9$ J kg^{-1}

$V_B = -7{\cdot}50 \times 10^8$ J kg^{-1}

$\Delta E_p = ?$

$\Delta V = -7{\cdot}50 \times 10^8 - (-1{\cdot}70 \times 10^9)$

$\Delta V = 9{\cdot}5 \times 10^8$ J kg^{-1}

$$\Delta E_p = \Delta V \times m$$

$\underline{\Delta E_p = 9{\cdot}5 \times 10^8 \times 1{\cdot}6 \times 10^3 = 1{\cdot}5 \times 10^{12}\ \text{J}}$

JOTTER NOTES

When a satellite changes altitude

Raise a satellite and r grows, so V and E_p both rise towards zero.

But a higher orbit is a slower orbit, so its E_k falls.

The energy you must supply is the rise in E_p minus the drop in E_k.

Not mandatory content — but the spec suggests considering it.

Starter — from memory

Close your notes. Write full sentences.

1. State the definition of gravitational potential.
2. Why is gravitational potential always negative?
3. Write the relationship for gravitational potential energy.
4. State the definition of gravitational field strength.
5. What is meant by a conservative field?

PERIOD 4 · ESCAPE

How fast to never come back?

Fast enough to reach infinity with nothing left over.

Escape velocity

JOTTER NOTES

LEARN THIS WORD FOR WORD

The escape velocity is the **minimum velocity** that allows a mass to escape a gravitational field **to infinity**, where the mass achieves zero kinetic energy and maximum (zero) potential energy.

Deriving $v_{esc} = \sqrt{(2GM/r)}$ — part 1

JOTTER NOTES

MANDATORY DERIVATION — you must reproduce every line

$$E_p = -\frac{GMm}{r}$$

Potential energy of the mass at distance r.

$$E_k = \frac{1}{2}mv^2$$

Kinetic energy it is given at launch.

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$

At infinity both are zero, so the total is zero.

Escape means arriving at infinity with nothing left over.

Deriving $v_{esc} = \sqrt{(2GM/r)}$ — part 2

JOTTER NOTES

MANDATORY DERIVATION — you must reproduce every line

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

Take the potential energy term across.

$$v^2 = \frac{2GM}{r}$$

Multiply by 2 and divide by m — the mass cancels.

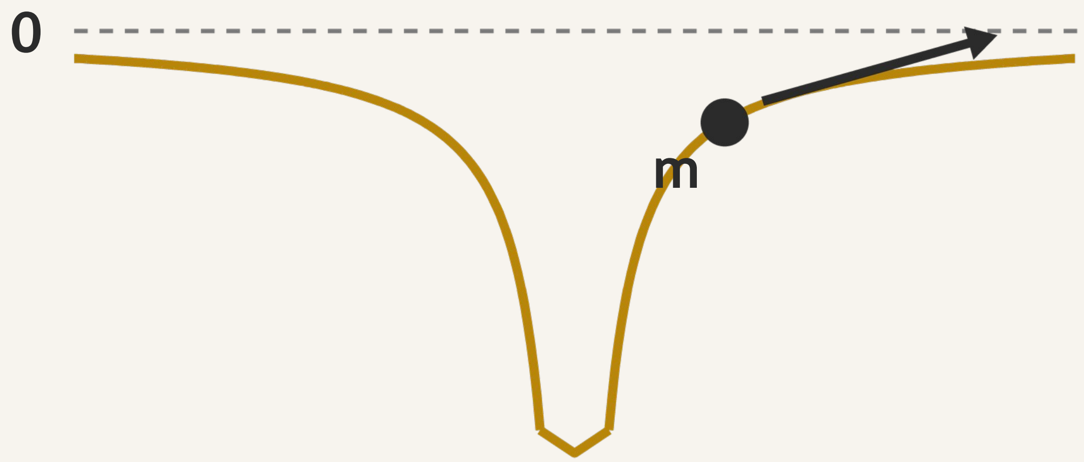
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

Square root both sides.

The escaping mass is not in the answer — only M and r.

The potential well

Escape velocity is the kinetic energy that reaches the top of the well



The dashed line is V = 0, reached only at infinity.

Worked example 5 — 2018 Q4(b)

An asteroid is $1{\cdot}58 \times 10^{11}$ m from the centre of HD 69830, mass $1{\cdot}7 \times 10^{30}$ kg. Find the minimum speed it needs to escape.

$r = 1{\cdot}58 \times 10^{11}$ m

$M = 1{\cdot}7 \times 10^{30}$ kg

$v_{esc} = ?$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$v_{esc} = \sqrt{\frac{2 \times 6 \cdot 67 \times 10^{-11} \times 1 \cdot 7 \times 10^{30}}{1 \cdot 58 \times 10^{11}}}$$

$\underline{v_{esc} = 3{\cdot}8 \times 10^{4}\ \text{m s}^{-1}}$

JOTTER NOTES

Why there is almost no helium in the air

Gas molecules in the atmosphere move at a range of speeds.

The lighter the molecule, the faster it moves at a given temperature.

Helium is light enough that some of it exceeds Earth's escape velocity and is lost to space; nitrogen and oxygen are not.

A planet's atmosphere follows from its escape velocity.

Gravitation in the exam

A question in every diet since 2016 — usually Q4

Diet	Question	What it asked
2016	Q3	Field strength and force
2017	Q4, Q5	Orbits; potential
2018	Q4	Satellite period; escape velocity
2019	Q5	Potential; change in E_p
2020	Q4, Q6c(ii)	Calculation
2022-25	Q4 each diet	Calculation

2018 and 2019 are worked in class above.

Reading and homework

Scholar: Rotational motion and astrophysics, Topic 5 — Gravitation

Scholar — Rotational motion, Topic 5
Gravitation
Scholar — end-of-topic test (homework)
BrightRed Study Guide pp. 22-31
Past papers — 2016, 2017, 2020, 2022-25

Sit the end-of-topic test for homework.

Scholar

Errors that cost marks

The five that come up every year

- r is measured between centres, not from the surface.
- G is a constant; g is a field strength. Not the same thing.
- The minus sign in $V = -GM/r$ is part of the answer.
- Escape velocity does not depend on the escaping mass.
- Convert AU and light-years to metres before substituting.

Check yourself

Can you do each of these without your notes?

1. State all three definitions, word for word.
2. Sketch the field lines for one mass, and for two.
3. Derive $v_{esc} = \sqrt{(2GM/r)}$, showing every step.
4. Find the mass of a star from a planet's orbit radius and period.
5. Explain why gravitational potential is always negative.

Experiments you could try

Standard kit, or your own phone

- Measure g with a pendulum: plot T^2 against length.
- Track the ISS with a phone app and time a full orbit.
- Use Stellarium for a planet's orbital radius and period.
- Model a potential well with a stretched sheet and a ball.
- Estimate the Moon's mass from its orbit radius and period.

Sources & credits

- Past-paper questions and marking instructions © Qualifications Scotland (SQA), reproduced for educational use.
- Notes, tests and homework: SCHOLAR Advanced Higher Physics, © Heriot-Watt University.
- Textbook: BrightRed Advanced Higher Physics Study Guide.
- All diagrams drawn for this deck; no images taken from the web.